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TECHNICAL REPORT NO. 414

AN APPROXIMATE ANALYSIS OF THE CONTINUUM LOW DENSITY HYPERSONIC FLOW OF A CHEMICALLY REACTING AND RADIATING GAS OVER A BLUNT NOSED BODY

By Massimo Trella
Roberto Vaglio-Laurin

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SUMMARY

The continuum low density hypersonic flow of a chemically reacting and radiating gas over a blunt body is analyzed by an approximate technique based on a recent alternative formulation of the well-known method of integral relations. Specifically, approximations are introduced for the streamwise variations of several flow properties, while detailed radial profiles, (consistent with the stipulated procedure) are sought by numerical analysis along normals to the body (essentially normals to streamlines); the particular formulation allows careful analysis of the large changes in physico-chemical properties of the gas between body and shock, as are to be encountered at the low Reynolds numbers where viscous effects are important over the entire shock layer. Detailed derivations of the simplified system of ordinary differential equations and of the boundary conditions governing the problem is presented. An outline of numerical procedures for applications is also given.

TABLE OF CONTENTS

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I. INTRODUCTION

The problem of observables associated with reentry of hypersonic vehicles has received considerable attention in recent years. This problem has strong physico-chemical as well as fluid mechanical overtones. Applications require detailed and accurate quantitative predictions of flow properties; such level of precision can only be obtained by numerical methods. A large body of related literature has recently been set forth; however, most analyses are pertinent to high Reynolds number flows where distinct shock, inviscid and boundary layer regions can be recognized. The range of applicability of those analyses is limited to altitudes below approximately 220 kft. for typical body dimensions (R=1 ft), and hypersonic reentry conditions. At the higher altitudes different flow regimes are encountered, which involve increasingly complex analyses. In an upward sweep of the trajectory, a continuum description of the flow can be applied to study the initial vorticity interaction, viscous layer, and incipient merged layer regimes while the range of lower Reynolds numbers must be analyzed on the basis of either transitional or free molecule flow theory.

The present study is concerned with the first three (continuum) regimes mentioned above. Even within these limits analysis becomes very cumbersome, particularly when effects of chemical reactions and radiation are included. Current state of the art permits detailed numerical investigation of either strictly inviscid flows (Ref. 2) or strictly boundary layer flows (Ref. 3) including coupled chemical reactions and, in preliminary fashion, radiation* (Ref. 4). Low density situations have been studied to limited extent and only in connection with flows of ideal gases; although related analyses rely heavily on extensive machine

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*For missile and lunar vehicle reentry, the effect of self absorption can become significant in some part of the shock layer (Ref. 4); however, in this preliminary study we assume the thin gas approximation to be valid over the entire flow field.

calculations results are only available for the stagnation point on a sphere (Ref. 5) and for flows about blunt bodies within the Newtonian approximation (Ref. 6). The limitations of those theories are well known.

The present analysis was undertaken having in mind the strict requirements for prediction of observables, definition of the range of possible observation, and interpretation thereof. However, in view of the exploratory nature of the task an approximate method of analysis has been sought, presumably retaining the essential features of the physical situation.

In the presence of viscous effects and of chemical reaction extending to the entire shock layer careful consideration must be given to distributions of state properties and composition in the direction normal to the body surface; on the basis of this consideration, we have adopted an integral method formulation based on approximate description of the distribution of flow properties in the direction parallel to the body surface (Refs. 7 and 8).

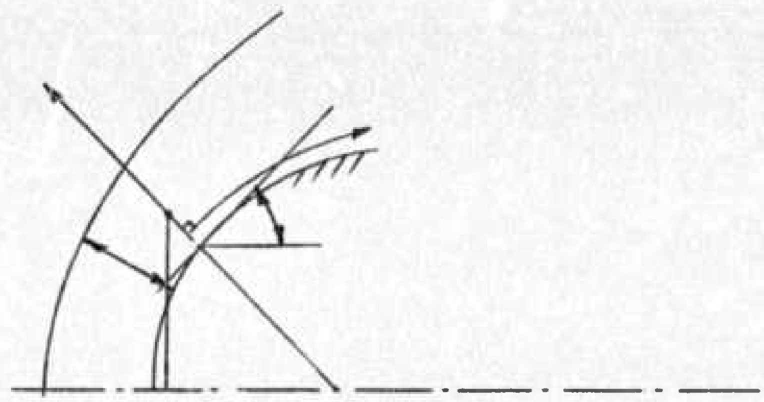
Details of the investigation are presented in the following sequence: Section II) The Governing Partial Differential Equations, Boundary Conditions and the Approximations Based on

Orders of Magnitude Analysis; Section III) The Integral Approximations and the Resulting Ordinary Differential Equations and Boundary Conditions; Section IV) Outline of Numerical Procedure Including Iterative Techniques Successfully Proven in Connection with other Problems.

The present analysis represents the first phase of a research program on reentry observables in low density continuum flows currently under way at GASL.

II. GOVERNING EQUATIONS AND BOUNDARY CONDITIONS

The equations governing the two-dimensional or axisymmetric flow of a viscous, chemically reacting and radiating gas, in a body oriented coordinate system (x,y,Φ) are:



Continuity:

$$\frac{\partial}{\partial x}\left(r^j \rho u\right) + \frac{\partial}{\partial y}\left(\chi r^j \rho v\right) = 0 \qquad (1)$$

Momentum x:

$$\rho\left\{u\frac{\partial u}{\partial x} + \chi v\frac{\partial u}{\partial y} + kuv\right\} = -\frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial}{\partial y}\left(\chi \tau_{xy}\right)$$
$$+ k\tau_{xy} + j\left\{\frac{\tau_{xx}-\tau_{\Phi\Phi}}{r}\frac{\partial r}{\partial x} + \frac{\chi \tau_{xy}}{r}\frac{\partial r}{\partial y}\right\} \qquad (2)$$

Momentum y:

$$\rho \left\{ u \frac{\partial v}{\partial x} + \chi v \frac{\partial v}{\partial y} - ku^2 \right\} = - \chi \frac{\partial p}{\partial y} + \chi \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + k(\tau_{yy} - \tau_{xx})$$

$$+ j \left\{ \frac{\tau_{xy}}{r} \frac{\partial r}{\partial x} + \chi \frac{(\tau_{yy} - \tau_{\Phi\Phi})}{r} \frac{\partial r}{\partial y} \right\} \tag{3}$$

Energy:

$$\rho \left\{ u \frac{\partial H}{\partial x} + \chi v \frac{\partial H}{\partial y} \right\} + \nabla \cdot \hat{q}_r = \frac{\partial}{\partial x} \left(\frac{1}{\chi} q_x \right) - \frac{\partial}{\partial y} (\chi q_y) + \frac{\partial}{\partial x} (u\tau_{xx} + v\tau_{xy})$$

$$+ \frac{\partial}{\partial y} \left[\chi (u\tau_{xy} + v\tau_{yy}) \right] - \frac{j}{r} \left\{ \left[\frac{1}{\chi} q_x \frac{\partial r}{\partial x} + \chi q_y \frac{\partial r}{\partial y} \right] + \right. \tag{4}$$

$$\left. + \left[(u\tau_{xx} + v\tau_{xy}) \frac{\partial r}{\partial x} + \chi (u\tau_{xy} + v\tau_{yy}) \frac{\partial r}{\partial y} \right] \right\}$$

Species:

$$\rho \left\{ u \frac{\partial \alpha_i}{\partial x} + \chi v \frac{\partial \alpha_i}{\partial y} \right\} = \frac{\partial}{\partial x} \left(\frac{\rho D}{\chi} \frac{\partial \alpha_i}{\partial x} \right) + \frac{\partial}{\partial y} \left(\rho D \chi \frac{\partial \alpha_i}{\partial y} \right)$$

$$+ \frac{j}{r} \left[\frac{\rho D}{\chi} \frac{\partial \alpha_i}{\partial x} \frac{\partial r}{\partial x} + \rho D \chi \frac{\partial \alpha_i}{\partial y} \frac{\partial r}{\partial y} \right] + \rho \chi \dot{w}_i \tag{5}$$

$$i = 1, 2 \ldots N-1$$

Where:

$$
\left.
\begin{aligned}
k &= \text{body curvature} \\
j &= \begin{cases} 0 & \text{two-dimensional flow} \\ 1 & \text{axisymmetric flow} \end{cases} \\
\chi &= 1 + ky \\
\Phi &= \text{azimuthal angle} \\
\tau_{xx} &= \frac{2\mu}{\chi}\left(\frac{\partial u}{\partial x} + kv\right) - \frac{2}{3}\mu\left[\frac{1}{\chi}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{kv}{\chi} + j\left(\frac{u}{\chi r}\frac{\partial r}{\partial x} + \frac{v}{r}\frac{\partial r}{\partial y}\right)\right] \\
\tau_{yy} &= 2\mu\frac{\partial v}{\partial y} - \frac{2}{3}\mu\left[\frac{1}{\chi}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{kv}{\chi} + j\left(\frac{u}{\chi r}\frac{\partial r}{\partial x} + \frac{v}{r}\frac{\partial r}{\partial y}\right)\right] \\
\tau_{\Phi\Phi} &= j\left[\frac{2\mu}{r}\left(\frac{u}{\chi}\frac{\partial r}{\partial x} + v\frac{\partial r}{\partial y}\right) - \frac{2}{3}\mu\left(\frac{1}{\chi}\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{kv}{\chi} + \frac{u}{\chi r}\frac{\partial r}{\partial x} + \frac{v}{r}\frac{\partial r}{\partial y}\right)\right] \\
\tau_{xy} &= \mu\left(\frac{1}{\chi}\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} - \frac{ku}{\chi}\right)
\end{aligned}
\right\} \quad (6)
$$

Furthermore for Le = Pr = 1 and optically thin gas:

$$\left.\begin{aligned} q_x &= -\mu \frac{\partial h}{\partial x} \\ q_y &= -\mu \frac{\partial h}{\partial y} \\ \nabla\cdot\hat{q}_r &= \chi 4\pi\tilde{\mu}\, B(T_s) \\ B(T_s) &= \frac{\sigma T^4}{\pi} \\ \rho D &= \frac{Le}{Pr}\,\mu = \mu \end{aligned}\right\} \quad (7)$$

The system of 4+N-1 equations is complemented by an equation of state in the form:

$$h = h(p,\rho,\alpha_i) \quad (8)$$

by a viscosity law:

$$\mu = \mu(p,\rho,\alpha_i) \quad (9)$$

and by the condition:

$$\sum_{i=1}^{N} \alpha_i = 1 \quad (10)$$

The corresponding unknowns in the original system are the two components of velocity u, v, the pressure p, the density ρ and (N-1) mass fractions.

Two sets of boundary conditions are imposed: one set at the body and one at the "outer edge" of the shock layer; we specify all the boundary conditions in a form appropriate for the "viscous layer regime," namely: 1) no slip and temperature jump phenomena are present at the surface of the body, and 2) the shock can be treated as a surface of discontinuity.

The boundary conditions at the body are:

$$\left.\begin{array}{l} u_b = v_b = 0 \\ h_b = h_b(x) \\ \text{Surface catalyticity: 1) if catalytic} \rightarrow \alpha_i \text{ in equilibrium} \\ \qquad\qquad \text{2) if not catalytic} \rightarrow \text{impermeability conditions for all species except surface material} \end{array}\right\} \quad (11)$$

The boundary conditions at the shock are obtained from the standard Rankine-Hugoniot relations in terms of free stream properties, of local shock inclination, and of a prescribed composition of the mixture on the downstream side; formally

$$\left.\begin{aligned} u &= u_s \\ v &= v_s \\ \rho &= \rho_s \\ p &= p_s \\ \alpha_i &= \alpha_{i_s} \end{aligned}\right\} \quad (12)$$

The total orde· of the original system is [7+2(N-1)]; the [7+2(N-1)] boundary conditions (11) and (12) are however insufficient since the shock geometry is also unknown. The difficulty is removed upon examination of the continuity equation at the body; this provides the additional boundary conditions, namely

$$(v_y)_b = 0 \qquad (11a)$$

The original system of Eqs. (1)-(5) may be simplified considerably for specific application to high Mach number flows. Under those conditions the orders of magnitude of the basic properties in the shock layer are:

$$\left.\begin{aligned}
&\frac{\rho_\infty}{\rho} \sim \epsilon << 1 \\
&v \sim \epsilon U_\infty \\
&u \sim U_\infty \\
&y \sim \delta \\
&\mu \sim \mu_s \\
&x \sim L >> \delta \\
&\chi \sim 1 \\
&k \sim L^{-1}
\end{aligned}\right\} \tag{13}$$

Accordingly the following simplified form of the equations is obtained when terms of order higher than $(\epsilon R_e)^{-1}$ are neglected (Ref. 5):

Continuity:

$$\frac{\partial}{\partial x}(r^j \rho u) + \frac{\partial}{\partial y}(\chi r^j \rho v) = 0$$

Momentum x:

$$\rho \left\{ u \frac{\partial u}{\partial x} + \chi v \frac{\partial u}{\partial y} + kuv \right\} = - \frac{\partial p}{\partial x} + \frac{\partial}{\partial y} \left(\chi \mu \frac{\partial u}{\partial y} \right)$$

Momentum y:

$$\rho \left\{u \frac{\partial v}{\partial x} + \chi v \frac{\partial v}{\partial y} - ku^2\right\} = - \chi \frac{\partial p}{\partial y} + \chi \frac{4}{3} \frac{\partial}{\partial y} \left(\mu \frac{\partial v}{\partial y}\right)$$

$$- \frac{2}{3} \chi \frac{\partial}{\partial y} \left(\frac{\mu}{\chi} \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial x} \left(\mu \frac{\partial u}{\partial y}\right)$$

$$+ j \left\{\frac{\mu}{r} \frac{\partial r}{\partial x} \frac{\partial u}{\partial y} - \chi \frac{\partial}{\partial y} \left(\frac{\mu}{\chi} \frac{u}{r} \frac{\partial r}{\partial x}\right)\right\} \tag{14}$$

Energy:

$$\rho \left\{u \frac{\partial H}{\partial x} + \chi v \frac{\partial H}{\partial y}\right\} = \frac{\partial}{\partial y} \left(\chi\mu \frac{\partial H}{\partial y}\right) - \chi 4\pi\tilde{\mu} B$$

Species:

$$\rho \left\{u \frac{\partial \alpha_i}{\partial x} + \chi v \frac{\partial \alpha_i}{\partial y}\right\} = \frac{\partial}{\partial y} \left(\chi\mu \frac{\partial \alpha_i}{\partial y}\right) + \chi\rho\dot{w}_i$$

The system (14) represents the basis of the present analysis. All subsequent developments are aimed at developing a method of solution. In this connection we begin by introducing new independent variables

$$s = x$$
$$n = y/\delta \tag{15}$$

with

$$\delta = \delta(s) \tag{16}$$

the shock detachment distance; the domain of integration is thus transformed into a rectangle.

Performing the transformation

$$\left.\begin{aligned}\frac{\partial}{\partial y} &= \frac{1}{\delta}\frac{\partial}{\partial n}\\ \frac{\partial}{\partial x} &= \frac{\partial}{\partial s} - \frac{n}{\delta}\frac{d\delta}{ds}\frac{\partial}{\partial n}\end{aligned}\right\} \tag{17}$$

and recasting the equations in divergence form (desirable for implementation of the integral approach) we obtain:

<u>Continuity</u>:

$$\frac{\partial}{\partial s}(r^j \rho u) - n\frac{\partial}{\partial n}\left(\frac{r^j}{\delta}\frac{d\delta}{ds}\rho u\right) + \frac{\partial}{\partial n}\left(\frac{r^j}{\delta}\chi\rho v\right) = 0$$

<u>Momentum x</u>:

$$\frac{\partial}{\partial s}\left[r^j(p+\rho u^2)\right] - n\frac{\partial}{\partial n}\left[\frac{r^j}{\delta}\frac{d\delta}{ds}(p+\rho u^2)\right] + \frac{\partial}{\partial n}\left(\frac{r^j}{\delta}\chi\rho uv\right)$$

$$- kr^j(jp\tan\theta_b - \rho uv) = \frac{\partial}{\partial n}\left(\frac{r^j}{\delta}\chi\tau_{xy}\right)$$

$$
\left.
\begin{aligned}
&\underline{\text{Momentum } y}: \\
&\frac{\partial}{\partial s}\left(r^j \rho u v\right) - n\frac{\partial}{\partial n}\left(\frac{r^j}{\delta}\,\frac{d\delta}{ds}\,\rho u v\right) + \frac{\partial}{\partial n}\left[\frac{r^j}{\delta}\,\chi\left(p+\rho v^2\right)\right] \\
&\qquad - k r^j\left[(1+j)p+\rho u^2\right] = \frac{\partial}{\partial n}\left(\frac{r^j}{\delta}\,\chi\tau_{yy}\right) + \frac{\partial}{\partial x}\left(r^j \tau_{xy}\right) \\
&\qquad - n\frac{\partial}{\partial n}\left(\frac{r^j}{\delta}\,\frac{d\delta}{ds}\,\tau_{xy}\right) \\
&\underline{\text{Energy}}: \\
&\frac{\partial}{\partial s}\left(r^j \rho u H\right) - n\frac{\partial}{\partial n}\left(\frac{r^j}{\delta}\,\frac{d\delta}{ds}\,\rho u H\right) + \frac{\partial}{\partial n}\left(\frac{r^j}{\delta}\,\chi \rho v H\right) \\
&\qquad = \frac{\partial}{\partial n}\left(\frac{r^j}{\delta}\,\chi\mu\,\frac{1}{\delta}\,\frac{\partial H}{\partial n}\right) - \chi r^j 4\pi\tilde{\mu}\,B \\
&\underline{\text{Species}}: \\
&\frac{\partial}{\partial s}\left(r^j \rho u \alpha_i\right) - n\frac{\partial}{\partial n}\left(\frac{r^j}{\delta}\,\frac{d\delta}{ds}\,\rho u \alpha_i\right) + \frac{\partial}{\partial n}\left(\frac{r^j}{\delta}\,\chi \rho v \alpha_i\right) \\
&\qquad = \frac{\partial}{\partial n}\left(\frac{r^j}{\delta}\,\chi\mu\,\frac{1}{\delta}\,\frac{\partial \alpha_i}{\partial n}\right) + \chi r^j \rho \dot{w}_i
\end{aligned}
\right\}
\tag{18}
$$

The boundary conditions are still expressed by (11) and (12) intended to apply at n=0 and at n=1 respectively.

III. THE INTEGRAL APPROACH

Methodology for solution of the simplified system of partial differential Eqs. (18) is established only for a particular formulation of the problem (the inverse problem where the shock shape is specified and the associated body is determined) and involves considerable labor. The practical problem of analyzing the flow about a given configuration can be solved within practical limits of numerical effort only in approximate fashion. A well known procedure to this effect is represented by the integral method in its various formulations (Refs. 7 and 8). In essence this method reduces the system of governing partial differential equations to an approximating system of ordinary differential equations by introducing assumed distributions of flow properties along one coordinate direction. In the present report we choose to make assumptions about the variation of flow properties in the streamwise direction since, in the presence of viscous effects and of chemical reactions extending over the entire shock layer, we are particularly interested in assessing the effect of transport phenomena on cross-stream distributions of physico-chemical properties of the gas.

We begin implementation of the aforenoted integral approach by integrating Eqs. (18) between two general stations s_1 and s_2

to obtain the integro-differential equations:

Continuity:

$$\left[r^j \rho u\right]_{s_1}^{s_2} - n \frac{\partial}{\partial n} \int_{s_1}^{s_2} \left(\frac{r^j}{\delta} \frac{d\delta}{ds} \rho u\right) ds + \frac{\partial}{\partial n} \int_{s_1}^{s_2} \left(\frac{r^j}{\delta} \chi \rho v\right) ds = 0 \tag{19a}$$

Momentum x:

$$\left[r^j (p+\rho u^2)\right]_{s_1}^{s_2} - n \frac{\partial}{\partial n} \int_{s_1}^{s_2} \left[\frac{r^j}{\delta} \frac{d\delta}{ds} (p+\rho u^2)\right] ds + \frac{\partial}{\partial n} \int_{s_1}^{s_2} \left(\frac{r^j}{\delta} \chi \rho uv\right) ds$$

$$- \int_{s_1}^{s_2} \left[kr^j (jp \tan \theta_b - \rho uv)\right] ds = \frac{\partial}{\partial n} \int_{s_1}^{s_2} \left(\frac{r^j}{\delta} \chi \tau_{xy}\right) ds \tag{19b}$$

Momentum y:

$$\left[r^j \rho uv\right]_{s_1}^{s_2} - n \frac{\partial}{\partial n} \int_{s_1}^{s_2} \left(\frac{r^j}{\delta} \frac{d\delta}{ds} \rho uv\right) ds + \frac{\partial}{\partial n} \int_{s_1}^{s_2} \left[\frac{r^j}{\delta} \chi (p+\rho v^2)\right] ds$$

$$- \int_{s_1}^{s_2} kr^j \left[(1+j)p+\rho u^2\right] ds = \frac{\partial}{\partial n} \int_{s_1}^{s_2} \left(\frac{r^j}{\delta} \chi \tau_{yy}\right) ds \tag{19c}$$

$$+ \left[r^j \tau_{xy}\right]_{s_1}^{s_2} - n \frac{\partial}{\partial n} \int_{s_1}^{s_2} \left(\frac{r^j}{\delta} \frac{d\delta}{ds} \tau_{xy}\right) ds$$

Energy:

$$\left[r^j \rho u H\right]_{s_1}^{s_2} - n\frac{\partial}{\partial n}\int_{s_1}^{s_2}\left(\frac{r^j}{\delta}\frac{d\delta}{ds}\rho u H\right)ds + \frac{\partial}{\partial n}\int_{s_1}^{s_2}\left(\frac{r^j}{\delta}\chi\rho v H\right)ds$$

$$= \frac{\partial}{\partial n}\int_{s_1}^{s_2}\left(\frac{r^j}{\delta}\chi\mu\frac{1}{\delta}\frac{\partial H}{\partial n}\right)ds - \int_{s_1}^{s_2}(\chi r^j 4\pi\tilde{\mu}\, B)ds \tag{19d}$$

Species:

$$\left[r^j \rho u \alpha_i\right]_{s_1}^{s_2} - n\frac{\partial}{\partial n}\int_{s_1}^{s_2}\left(\frac{r^j}{\delta}\frac{d\delta}{ds}\rho u \alpha_i\right)ds + \frac{\partial}{\partial n}\int_{s_1}^{s_2}\left(\frac{r^j}{\delta}\chi\rho v \alpha_i\right)ds$$

$$= \frac{\partial}{\partial n}\int_{s_1}^{s_2}\left(\frac{r^j}{\delta}\chi\mu\frac{1}{\delta}\frac{\partial \alpha_i}{\partial n}\right)ds + \int_{s_1}^{s_2}(\chi r^j \rho \dot{w}_i)ds \tag{19e}$$

Reduction of the (3+N) Eqs. (19) to ordinary differential form follows immediately upon stipulation of the s-distribution of certain flow quantities. We point out that only (3+N) quantities can be subject of independent description; of these, two must be velocities, two state properties, and (N-1) species. We assume polynomial laws for the following:

$$\left.\begin{aligned}
\rho u &= A_1 s + A_3 s^3 + \cdots \\
\rho u^2 &= B_2 s^2 + B_4 s^4 + \cdots \\
\rho uv &= C_1 s + C_3 s^3 + \cdots \\
\rho uh &= D_1 s + D_3 s^3 + \cdots \\
\rho u\alpha_i &= E_{1i} s + E_{3i} s^3 + \cdots
\end{aligned}\right\} \tag{20}$$

where the coefficients A, B, C, D, E are functions of n. The selected quantities are all equal to zero at the axis and the assumed s dependencies satisfy the symmetry conditions; therefore, the number of terms in each polynomial is equal to the number of control stations away from the axis. Consistently with this procedure the properties at the axis are found from ratios of the laws (20) and not from the independent solution of the stagnation point partial differential equations (Ref. 5); for example:

$$\left[v(n)\right]_{\text{axis}} = \left[\frac{uv}{u}\right]_{\text{axis}} = \frac{C_1(n)}{A_1(n)} \tag{21}$$

and so on for all other properties. We also notice that in the case of one control station and one term in the laws (20), the proposed procedure is equivalent to assuming flow similarity.

In addition to the four independent quantities (20) other accessory properties are cast in a polynomial form to permit a more convenient numerical integration of the final system; naturally the degree of these polynomials is determined by the constitutive relations to be satisfied by the quantities on hand [e.g. Eqs. (6)] consistent with the postulated distributions (20). Thus, we set

$$\begin{aligned}
\rho u \tau_{xy} &= F_1 s + F_3 s^3 + \cdots + F_{m_1} s^{m_1} \\
\rho u \tau_{yy} &= G_1 s + G_3 s^3 + \cdots + G_{m_2} s^{m_2} \\
\rho u \mu \frac{\partial H}{\partial n} &= L_1 s + L_3 s^3 + \cdots + L_{m_3} s^{m_3} \\
\rho u \mu \frac{\partial \alpha_i}{\partial n} &= M_{i1} s + M_{i3} s^3 + \cdots + M_{i_{m_4}} s^{m_4}
\end{aligned} \tag{22}$$

consistent with symmetry requirements and specific definitions. Upon substitution of (20) into (19) and (22) one obtains t simultaneous nonlinear ordinary differential equations of the first order which can be cast in the form

$$\sum_{\lambda=1}^{t} a_{\sigma\lambda} \frac{dP_\lambda}{dn} = F_\sigma \tag{23}$$

where

$$\sigma\ \lambda = 1,2,3...t$$

t is defined in terms of the number z of control stations (s = const. = s_1, s_2...), the number of chemical species N and the magnitude of the highest powers m_i appearing in (22)

$$t = z(3+N) + m_1 + m_2 + m_3 + (N-1)m_4 \qquad (23a)$$

$P_\lambda(n)$ denotes the general coefficient (A_i, B_i, etc.) in the stipulated distributions (20) and (22), $a_{\sigma\lambda}$ and F_σ are known functions of s_i, n and P_λ to be determined case by case in accord with the number z of stations considered. Of the t Eqs. (23), z(3+N) follow from the conservation laws (19), while the remaining $[m_1+m_2+m_3+(N-1)m_4]$ are obtained by setting to zero the coefficients of all powers of s in (22). The t boundary conditions for the system (23) may readily be obtained in the following way: z(3+N) conditions are obtained by simultaneous consideration of (11), (12) and (20); $[m_1+m_2+m_3+(N-1)m_4]$ additional conditions are derived therefrom by substitution into (22) and subsequent setting to zero of the coefficients of all powers of s. Recognition of additional z unknowns, namely the z coefficients in the law describing the shock shape, does not alter the order of the system and merely requires implementation of the conditions (11a) at the z control stations.

In summary, an integral method analysis of the viscous flow of a mixture of N reactant gases over either two-dimensional or axisymmetric bodies, by recognizing z regions (strips) within the domain of interest, involves the solution of a system of t ordinary differential equations of first order (23) plus z algebraic equations, subject to t boundary conditions [t defined by (23a)]. The detailed derivation of expressions for coefficients and forcing functions in the system must be carried out case by case (depending upon the selected number z and the thermodynamic behavior of the mixture); however, some rules of internal consistency must be respected in that process as described below. To be consistent with the stipulation (20) various flow properties of interest must be described by the relations:

$$\rho v = \frac{(\rho u v)(\rho u)}{(\rho u^2)} \tag{24a}$$

$$\rho v^2 = \frac{(\rho u v)^2}{(\rho u^2)} \tag{24b}$$

$$\alpha_i = \frac{(\rho u \alpha_i)}{(\rho u)} \tag{24c}$$

$$h = \frac{(\rho uh)}{(\rho u)} \tag{24d}$$

$$H = h + \frac{1}{2}(u^2+v^2) = h + \frac{1}{2}\frac{(\rho u^2)}{(\rho u)^2}\left[(\rho u^2)+(\rho v^2)\right] \tag{24e}$$

$$p = \rho h\pi(s,n) = \frac{(\rho uh)(\rho u)}{(\rho u^2)}\pi(s,n) \tag{24f}$$

The formal expression for the pressure p is suggested by consideration of ideal gas flows wherein π = const; in the more general case one has

$$\pi(s,n) = \pi(\rho,h,\alpha_i) \tag{25}$$

As an example of consistent derivation let us examine a complicated term such as that involving the pressure in the x-momentum Eq. (19b):

$$\frac{\partial}{\partial n}\int_{s_1}^{s_2}\frac{r^j}{\delta}\frac{d\delta}{ds}\,p\,ds \tag{26}$$

The consistent law for the shock is

$$\delta = \delta_0 + \delta_1 s^2 + \delta_2 s^4 + \cdots \tag{27}$$

Thus

$$\frac{r^j}{\delta}\frac{d\delta}{ds} = R_0(s) + jn\,R_1(s) \tag{28}$$

with

$$\left.\begin{aligned} R_0(s) &= \frac{1}{\delta}\ \frac{d\delta}{ds}\left(\frac{\cos\ \theta_b}{k}\right)^j \\ R_1(s) &= \frac{d\delta}{ds}\cos\ \theta_b \end{aligned}\right\}\quad (29)$$

The appropriate sequence of transformations of (26) then proceeds along the following lines:

$$\frac{\partial}{\partial n}\int_{s_1}^{s_2}\frac{r^j}{\delta}\ \frac{d\delta}{ds}\ p\ ds = \frac{\partial}{\partial n}\int_{s_1}^{s_2}\Big[R_0(s)+jn\ R_1(s)\Big]\times$$

$$\times\left[\frac{A_1 D_1+(D_3 A_1+A_3 D_1)s^2+D_3 A_3 s^4}{B_2+B_4 s^2}\right]\pi(s,n)ds$$

$$=\frac{\partial}{\partial n}\Big\{A_1 D_1\int_{s_1}^{s_2}\frac{R_0(s)+jn\ R_1(s)}{B_2+B_4 s^2}\ \pi(s,n)ds$$

$$+\ (D_3 A_1+A_3 D_1)\int_{s_1}^{s_2}\frac{R_0(s)+jn\ R_1(s)}{B_2+B_4 s^2}\ s^2\pi(s,n)ds$$

$$+\ D_3 A_3\int_{s_1}^{s_2}\frac{R_0(s)+jn\ R_1(s)}{B_2+B_4 s^2}\ s^4\pi(s,n)ds\Big\}$$

$$=\frac{\partial(A_1 D_1)}{\partial n}\int_{s_1}^{s_2}\frac{R_0(s)+jn\ R_1(s)}{B_2+B_4 s^2}\ \pi(s,n)ds$$

$$+ \frac{\partial(D_3A_1+A_3D_1)}{\partial n}\int_{s_1}^{s_2}\frac{R_0(s)+jn\,R_1(s)}{B_2+B_4s^2}\,s^2\pi(s,n)ds$$

$$+ \frac{\partial(D_3A_3)}{\partial n}\int_{s_1}^{s_2}\frac{R_0(s)+jn\,R_1(s)}{B_2+B_4s^2}\,s^4\pi(s,n)ds$$

$$- \frac{\partial B_2}{\partial n}\Big\{A_1D_1\int_{s_1}^{s_2}\frac{R_0(s)+jn\,R_1(s)}{(B_2+B_4s^2)^2}\,\pi(s,n)ds$$

$$+ (D_3A_1+A_3D_1)\int_{s_1}^{s_2}\frac{R_0(s)+jn\,R_1(s)}{(B_2+B_4s^2)^2}\,s^2\pi(s,n)ds$$

$$+ D_3A_3\int_{s_1}^{s_2}\frac{R_0(s)+jn\,R_1(s)}{(B_2+B_4s^2)^2}\,s^4\pi(s,n)ds\Big\}$$

$$- \frac{\partial B_4}{\partial n}\Big\{A_1D_1\int_{s_1}^{s_2}\frac{R_0(s)+jn\,R_1(s)}{(B_2+B_4s^2)^2}\,s^2\pi(s,n)ds \tag{30}$$

$$+ (D_3A_1+A_3D_1)\int_{s_1}^{s_2}\frac{R_0(s)+jn\,R_1(s)}{(B_2+B_4s^2)^2}\,s^4\pi(s,n)ds$$

$$+ D_3A_3\int_{s_1}^{s_2}\frac{R_0(s)+jn\,R_1(s)}{(B_2+B_4s^2)^2}\,s^6\pi(s,n)ds\Big\}$$

$$+ j\int_{s_1}^{s_2}R_1(s)\,\frac{A_1D_1+(D_3A_1+A_3D_1)s^2+D_3A_3s^4}{B_2+B_4s^2}\,\pi(s,n)ds$$

$$+ \int_{s_1}^{s_2}\Big[R_0(s)+jn\,R_1(s)\Big]\frac{A_1D_1+(D_3A_1+A_3D_1)s^2+D_3A_3s^4}{B_2+B_4s^2}\,\frac{\partial\pi}{\partial n}(s,n)ds$$

All the integrals in (30) except the last one may be evaluated in straightforward fashion to obtain associated contributions to the coefficients $a_{\sigma\lambda}$; the last integral is reduced to the desired form by means of the additional relationship:

$$p = \left[R_0(s)+jn\ R_1(s)\right] \frac{A_1 D_1 + (D_3 A_1 + D_1 A_3) s^2 + D_3 A_3 s^4}{B_2 + B_4 s^2} \tag{31}$$

and transformation:

$$\int_{s_1}^{s_2} p \frac{\partial\pi}{\partial n} ds = \int_{s_1}^{s_2} p \left\{ \frac{\partial(\rho u)}{\partial n} \left[- \frac{(\rho uh)}{(\rho u)^2} \frac{\partial\pi}{\partial h} - \frac{(\rho u\alpha_i)}{(\rho u)^2} \frac{\partial\pi}{\partial\alpha_i} + \frac{(\rho u)}{(\rho u^2)} \frac{\partial\pi}{\partial\rho} \right] \right.$$

$$+ \frac{\partial(\rho uh)}{\partial n} \frac{1}{\rho u} \frac{\partial\pi}{\partial h} + \frac{\partial(\rho u\alpha_i)}{\partial n} \frac{1}{\rho u} \frac{\partial\pi}{\partial\alpha_i}$$

$$\left. - \frac{\partial(\rho u^2)}{\partial n} \frac{(\rho u)^2}{(\rho u^2)^2} \frac{\partial\pi}{\partial\rho} \right\} ds$$

$$= \frac{dA_1}{dn} \int_{s_1}^{s_2} p_s \left[- \frac{(\rho uh)}{(\rho u)^2} \frac{\partial\pi}{\partial h} - \frac{(\rho u\alpha_i)}{(\rho u)^2} \frac{\partial\pi}{\partial\alpha_i} + \frac{(\rho u)}{(\rho u^2)} \frac{\partial\pi}{\partial\rho} \right] ds$$

$$+ \frac{dA_3}{dn} \int_{s_1}^{s_2} p_s{}^3 \left[- \frac{(\rho uh)}{(\rho u)^2} \frac{\partial\pi}{\partial h} - \frac{(\rho u\alpha_i)}{(\rho u)^2} \frac{\partial\pi}{\partial\alpha_i} + \frac{(\rho u)}{(\rho u^2)} \frac{\partial\pi}{\partial\rho} \right] ds$$

$$+ \frac{dD_1}{dn} \cdots\cdots + \tag{32}$$

All coefficients and forcing functions in (23) may consistently be determined by algebraic transformations as detailed at (26) through (32) above.

Solution of the system (23) may only be sought by numerical means; proper formulation thereof requires a preliminary assessment of the general features of the integral curves to be obtained. In this connection we observe that the integral formulation adopted here does not exhibit critical points of the type present in the standard integral method; hence the integration is straightforward. The absence of singularities follows from the a priori assumed regular behavior at the sonic velocity as manifested either by the prescribed pressure distribution on the body or by the prescribed shock shape (see the alternative procedures in Section IV). The singularity in the standard formulation represents a mathematical manifestation of the continuity requirement; in that case we start by assuming a shock detachment distance δ_0 at the axis and compute increment thereof. Here we do not say anything about δ; it is completely free to satisfy the continuity requirement consistent with the assumed regular behavior.

Consideration of viscous effects, specifically the no-slip condition at the wall, in the present formulation renders the equations singular at n=0; however, this singularity is regular and integration in its neighborhood may readily be performed along the lines detailed in Ref. 5.

IV. OUTLINE FOR NUMERICAL PROCEDURE

In analogy with the inviscid blunt body problem two possibilities are open for the problem on hand: the direct and the inverse method.

Analysis by the direct method proceeds along the following lines: a) the body geometry and the body temperature are given; b) (N+3) distributions of flow properties (polynomials in s) are estimated at n=0; these are, for example, the shock geometry $\delta(s)$, the pressure p, the shear stress [whereby a relation between $(\rho u)_n$ and $\left[\frac{(\rho u)^2}{\rho u^2}\right]_n$ is obtained], and either the composition if the wall is not catalytic, or the normal derivative of the N species concentration $\left(\frac{\rho u \alpha_i}{\rho u}\right)_n$ if the wall is catalytic; c) the remaining inputs for the integration are consistently obtained from the study of the equations in the neighborhood of the body; specifically, one requires $v_n=0$ to satisfy continuity and one determines p_n to satisfy the y momentum equation consistent with the prescribed p(s)*; d) numerical integration of (23) is performed from n=0 to n=1; e) compliance of the results of integration at n=1 with the boundary (jump) conditions for ρ, p, u, v and α_i is

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*The aforenoted set of inputs is equivalent to prescribing the heat transfer since $h_n=h_n(p_n,\rho_n,\alpha_{in})$.

inspected; and f) if the latter conditions are not satisfied initial estimated values are modified and an iterative solution performed following criteria such as are discussed later in this section.

Analysis by the inverse method proceeds along the following lines: a) the shock geometry is given; b) (4+N) distributions of flow properties (e.g. $\delta, u_n, p_n, h_n, \alpha_{in}$) are estimated and prescribed at the outer edge of the shock layer (n=1) and initial values for integration established accordingly; c) integration is carried out from n=1 to n=0; d) compliance with (4+N) conditions on the body (namely $u=v=v_n=0$, $h=h_b$ and either α_i or α_{in} consistent with surface catalyticity) is inspected; and e) iteration is pursued if not all conditions are satisfied at n=0.

It must be concluded that either method exhibits comparable numerical difficulties for the problem on hand; indeed a large number of quantities [either (3+N) or (4+N)] must be estimated and iterated upon for either direct or inverse approach.

The required iterations may be carried out in organized fashion by assuming linearization of differences between solutions characterized by change in one input parameter only. To be specific, a first run is performed corresponding to an initial set of (3+N) estimated quantities; subsequently (3+N) additional runs are performed each characterized by a set of initial conditions differing from the first one for a single quantity. In the hypothesis that the error can be linearized a system of linear algebraic equations in the unknown ΔQ_j $[j=1,2,\cdots(3+N)]$ corrections to be imposed on the estimated initial conditions may be constructed; specifically one equates the errors observed in the first run to a weighted linear combination of the errors introduced by each perturbation to obtain

$$P_i = \sum_{j=1}^{3+N} \frac{\partial P_i}{\partial Q_j} \Delta Q_j \tag{33}$$

where P_i indicates the known initial error property for the i^{th} property and the partial derivatives are obtained by differences between the results of each "perturbation" run and the initial one. Obviously the procedure can be repeated until satisfactory convergence is achieved.

A second alternative to the numerical iteration may be sought by extension to the problem on hand of Weil's method for solving a class of ordinary differential equations usually encountered in boundary layer problems. The essence of that method is the approximate solution of the equations by iteration, satisfying exactly at each iteration the known boundary conditions. Although the characteristics of convergence can not be predicted, an approach along this line is here suggested in view of its simplicity, its adaptability to automatic operation and its capability to provide results to any degree of accuracy. The salient procedural points are clearly illustrated by inspection of the following model solution of a system of two non-linear ordinary differential equations of the first order; extension to the higher order system (25) is immediate. Consider the equations

$$\begin{aligned} a_{11}\frac{df_1}{dx} + a_{12}\frac{df_2}{dx} &= F_1 \\ a_{21}\frac{df_1}{dx} + a_{22}\frac{df_2}{dx} &= F_2 \end{aligned} \qquad (34)$$

with a_{11}, a_{12}, a_{21}, a_{22}, F_1, F_2 = known functions of f_1, f_2, x, subject to the boundary conditions

$$\begin{aligned} &\text{at} \quad x = 0 \quad f_1 = f_{10} \\ &\text{at} \quad x = 1 \quad f_2 = f_{21} \end{aligned} \qquad (35)$$

Solve the system (34) for the highest order derivatives (in this case first order):

$$\frac{df_1}{dx} = \frac{F_1 - \frac{a_{12}}{a_{22}} F_2}{a_{11} + a_{21} \frac{a_{12}}{a_{22}}}$$

$$\frac{df_2}{dx} = \frac{1}{a_{12}} \left[F_1 - a_{11} \frac{df_1}{dx} \right] \tag{36}$$

At the general n^{th} iteration the properties $(f_1)_{n-1}$ and $(f_2)_{n-1}$ are known (in the initial stage estimated profiles satisfying the known boundary conditions are used as inputs. Write Eq. (36) in the form:

$$\left(\frac{df_1}{dx}\right)_n = \left[\frac{F_1 - \frac{a_{12}}{a_{22}} F_2}{a_{11} + a_{21} \frac{a_{12}}{a_{22}}} \right]_{n-1}$$

$$\left(\frac{df_2}{dx}\right)_n = \left[\frac{1}{a_{12}} \left(F_1 - a_{11} \frac{F_1 - \frac{a_{12}}{a_{22}} F_2}{a_{11} + a_{21} \frac{a_{12}}{a_{22}}} \right) \right]_{n-1} \tag{37}$$

Upon integration

$$[f_1(x)]_n = f_{1o} + \int_o^x \left[\frac{F_1 - \frac{a_{12}}{a_{22}} F_2}{a_{11} + a_{21} \frac{a_{12}}{a_{22}}}\right]_{n-1} dx$$

$$[f_2(x)]_n = (f_{2o})_n + \int_o^x \left[\frac{1}{a_{12}} \left(F_1 - a_{11} \frac{F_1 - \frac{a_{12}}{a_{22}} F_2}{a_{11} + a_{21} \frac{a_{12}}{a_{22}}}\right)_{n-1}\right] dx \tag{38}$$

The boundary conditions at x=1 define

$$(f_{2o})_n = f_{21} - \int_o^1 \left[\frac{1}{a_{12}} \left(F_1 - a_{11} \frac{F_1 - \frac{a_{12}}{a_{22}} F_2}{a_{11} + a_{21} \frac{a_{12}}{a_{22}}}\right)_{n-1}\right] dx \tag{39}$$

and, therefore, Eqs. (38) can be used to generate the new profiles. The procedure is with a limit on $\frac{\left[(f_{2o})_n - (f_{2o})_{n-1}\right]}{(f_{2o})_n}$. The procedure is amenable to further refinements: in particular when the solution oscillates an over-relaxed iteration or extrapolation can be used to expedite convergence.

V. CONCLUSIONS

A method has been presented for the study of the continuum viscous shock layer about re-entry bodies as it is encountered at altitudes approximately between 300 and 200 kft for vehicles having typical dimensions of the order of feet. An integral approach has been used which permits detailed description of distributions of flow properties along normals to the body surface and, thereby, should lead to reasonable predictions of observables. The present report has been concerned with the initial phase of the investigation, namely the development of the method of analysis. Applications will require extensive numerical work; guide lines therefor have been discussed.

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